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On the Measures of Possibility and Probability of Uncertainty: A Fuzzy Theoretical Approach

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ABSTRACT: Uncertainty remains one of the most fundamental challenges in science, philosophy, and artificial intelligence (AI). Classical probability theory provides a means to quantify randomness, while possibility theory offers a way to describe vagueness and incomplete information. This paper explores the theoretical measures of possibility and probability within the fuzzy theoretical framework, focusing on their conceptual distinctions, mathematical relationships, and integrative potential. Drawing on developments in fuzzy set theory, this study recognizes the complementary roles of possibility and probability measures in modelling uncertainty and decision-making systems. It therefore, proposes a hybrid uncertainty representation that unites probabilistic and possibilistic reasoning. Applications in environmental risk analysis and decision systems demonstrate how fuzzy theory that bridges the gap between quantitative and qualitative uncertainty. The results contribute to a unified understanding of uncertainty representation and reasoning under imprecise conditions. The analysis highlights the implications of these measures of uncertainty in reasoning, risk assessment and intelligent system design.

KEYWORDS: Fuzzy logic, possibility theory, probability theory, uncertainty modeling, epistemic uncertainty, risk analysis

1. INTRODUCTION

Uncertainty is inherent in both natural and artificial systems. It influences decision-making processes in diverse fields, including climate science, finance, and AI. Classical approaches to uncertainty, particularly probability theory have long served as the standard for representing randomness in scientific and engineering analysis. Traditionally, probability theory has been the dominant framework for quantifying uncertainty by modeling randomness in observable events (Kolmogorov, 1933, Jaynes, 2003). However, this probability theory approach assumes the availability of complete statistical information, which may not hold in many real-world systems characterized by vagueness and partial knowledge. It stands, therefore that the assumption of a well-defined set of outcomes and precise data, are often unavailable in real-world applications.

In contrast, possibility theory, introduced by Zadeh (1978), captures vagueness and partial truth by representing degrees of plausibility rather than frequencies. This form of uncertainty—known as epistemic uncertainty—stems from incomplete or imprecise knowledge rather than random variation. Dubois and Prade (1988) expanded on Zadeh's theory, formalizing it as a complementary framework to probability theory.

While probability measures randomness, possibility quantifies imprecision, the integration of both within fuzzy logic offers a comprehensive means of reasoning about uncertainty (Zimmermann, 2010). This study revisits these measures, examines their mathematical properties, and proposes a fuzzy-theoretical approach that unifies them within a single coherent framework.

To address these limitations, Zadeh (1978) introduced possibility theory as a framework for dealing with imprecise and qualitative information. Possibility theory, grounded in fuzzy set theory, allows for the representation of uncertainty in systems where probabilistic information is either unavailable or inadequate. This theoretical paper examines the conceptual and mathematical relationships between probability and possibility measures, their integration within fuzzy logic, and their practical significance for modeling uncertainty across scientific and AI domains.

From the literature, possibility theory considered to handle vague ad imprecise information in linguistic statements. Here we present a basic background and in section two we introduce possibility distribution along with probability and necessity measures that are computed from the distributions. In section three we described the method used in the study and then explained the main results in section four, which touches on probability-possibility transformation that suits our objectives. It is important to observe that the possibilistic framework has led to many useful results in classification problems, where classifiers have combined possibility distributions using Tnorm functions especially AI research.

2. THEORETICAL BACKGROUND AND LITERATURE REVIEW

2.1 The Nature of Uncertainty

Uncertainty arises from two principal sources: aleatory uncertainty, due to inherent randomness, and epistemic uncertainty, resulting from incomplete knowledge (Aven & Zio, 2018). Probability theory is suited for the former, while possibility theory is more appropriate for the latter. Most real-world systems—such as environmental hazards or socio-economic forecasting—exhibit both, necessitating hybrid models that integrate probabilistic and possibilistic reasoning (Dubois, 2011).

In general, uncertainty arises from incomplete information, variability, or ambiguity about the state of a system. Four fundamental forms of uncertainty are often recognized:

1. Aleatory uncertainty — This refers to randomness or inherent variability (e.g., noise in data).
2. Epistemic uncertainty — This is associated with lack of knowledge or incomplete understanding about system parameters. (Dubois, 2011)
3. Ontological uncertainty — This is linked to indeterminacies in the structure of reality or unknown unknowns in the model's structure.
4. Linguistic uncertainty — This concerns ambiguity in language or qualitative descriptions or interpretation of information.

Uncertainty has been interpreted differently by three major epistemological schools of thought or views. From a philosophical standpoint, these dominant views shape the understanding of uncertainty:

- The *positivist* view, which considers uncertainty as measurable and objective, emphasizing statistical laws and empirical verification.
- The *constructivist* perspective, which views uncertainty as subjective and socially constructed, dependent on context and interpretation (Aven & Zio, 2018).
- The *pragmatist* approach, which reconciles these extremes by focusing on how uncertainty can be effectively modeled and managed in practice.

Our study suggests that fuzzy logic provides a unifying framework that transcends these perspectives by acknowledging that truth and membership can exist in degrees rather than binary absolutes. It supports reasoning under partial information, which is closer to human cognition and decision-making. Thus, fuzzy theory situates itself between objective quantification (probability) and subjective interpretation (possibility).

2.2 Historical Background

According Agarwal and Nayal (2015), uncertainty-based information was first conceived in terms of classical set theory and in terms of probability theory. Information theory as developed in 1948 by Claude I. Shannon, has been based on and understood from the perspective of measure theory of probabilistic uncertainty (Channon and Beaver, 1949). However, it was Kolmogorov (1933) who earlier, had developed the axiomatization of probability as the mathematical foundation for uncertainty arising from stochastic processes. In contrast, Zadeh (1965) introduced fuzzy set theory to handle linguistic vagueness, which he later extended into possibility theory (Zadeh, 1978) as a complementary framework to probability theory, which is capable of epistemic uncertainty that arises from incomplete information rather than stochastic variability. Dubois and Prade (1988) further demonstrated that possibility theory can model uncertainty without requiring complete statistical information, offering an alternative reasoning mechanism for human-like perception and decision-making. Again, through the generalized information theory, which gained traction in the 1980s, the focus has been to capture properties of uncertainty-based information formulated within any feasible mathematical framework. This has resulted in the understanding of uncertainty-based information in addition to classical set theory, fuzzy set theory, possibility theory along with evidence theory, plausibility theory in a more nuanced way.

2.3 Measures of Probability and Possibility

- **Probability Measure**

Possibility theory is a measure-based framework that assigns a value between 0 and 1 to each event satisfying the additive axiom. Probability theory quantifies uncertainty by assigning real numbers to events, satisfying axioms of additivity ad normalization. A probability measure P, is accordingly defined over a sigma algebra F over a sample space Ω, which assigns a real number to each event that it satisfies. For the sample space Ω, the probability measure:

$$\mathrm{P}: \mathrm{P}(\Omega) \to (0,1) \text{ satisfies}$$
$$P(\Omega) = 1,$$
$$\mathrm{P}(\theta) = 0, and$$
$$\mathrm{P}(A \vee B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \wedge B), \forall A, B \subseteq \Omega$$

- **Fuzzy set and Possibility Measure**

Fuzzy set theory represents imprecise concepts by defining a membership function

$$\pi_A(\mathrm{x}): \mathrm{X} \to [0,1],$$

describing the degree to which x belongs to fuzzy set X in a fuzzy set A. Building upon this x, Zadeh (1978) introduced the possibility distribution π_A, from which the possibility measure $\Pi(A)$ and necessary measure $\aleph(A)$ are derived

A *possibility measure* Π, is defined through a possibility distribution $\mu(x)\epsilon[0,1]$ over Ω, which expresses how plausible each element x is within the universe Ω, (Zadeh, 1978). For any event $A, B \subseteq \Omega$:

$$\Pi(A) = sup_{x\epsilon A}\pi(x), \aleph(A) = 1 - \Pi(A^c),$$
$$\Pi(A\ V\ B) = max(\Pi(A), \Pi(B))$$

A dual, the *necessity measure* complements possibility as:

$$N(A) = 1 - \pi(A^c)$$

Probability distribution is a total of 1 across all possible outcomes. Possibility on the other, ensures that at least one event is fully possible.

While probability is derived from frequencies or likelihoods, possibility arises from membership functions within fuzzy sets. A fuzzy membership function expresses the degree of belonging of an element to a concept or event.

A key distinction is *interpretative*: probability quantifies how likely something (e.g. an event) is to occur, while possibility quantifies how consistent something is with available knowledge (Dubois & Prade, 1988). Intermediate models, such as credibility measures (Liu, 2004), attempt to balance both frameworks, especially in systems requiring both stochastic and epistemic reasoning.
For instance, in weather forecasting, the probability of rain may be 0.6 (based on statistics), but the possibility may be 0.9 (based on fuzzy linguistic descriptors such as "very cloudy").

3. METHODOLOGY

The study employs a mathematical-analytical approach to model the interrelation between possibility and probability. The main idea is to construct a mapping operator T that transforms a possibility distribution $\pi(x)$ into a probability density function $\rho(x)$ while maintaining logical consistency.

3.1 Defining the mapping function

Let X be a continuous domain with a normalized possibility distribution such that:

$$sup_{x\epsilon X}\pi(x) < 1,$$

$\rho(x) < T\big(\pi(x)\big)$, and $\int_x \rho(x)dx < 1$,

$$P(A) = \int_x T\big(\pi(x)\big)dx$$

Monotonicity: Let π_1 and π_2 be arbitrary assertions on the π distribution (i.e. $\pi_1, \pi_2 \subseteq \pi$)
If $\pi_1(x) \leq \pi_2(x)$, then $T(\pi_1(x)) \leq T(\pi_2(x))$
Normalization: ensures that total probability equals 1

$$\sum_{x\epsilon A} \rho(x) = 1$$

This is such that x is uniquely represented by probability density function $\mathrm{P}: \mathrm{X} \rightarrow [0,1]$ though the formula

$$\pi(A) = \max \pi(A)$$

Coherence: maintains order consistency between possibility and probability rankings

3.2 Analytical Framework

We proposed that T a be modelled as an exponential or power function: $T(\pi_1(x)) = \frac{\pi(x)^\alpha}{\int_x \pi(x)^\alpha dx}$

For α =1, the transformation preserves proportional for higher α, it emphasizes higher membership degrees, approaching crisp probabilistic distributions.

3.3 Model Representation and Comparison of Measures

The two measures differ fundamentally in interpretation but can complement each other when modeled within a fuzzy framework (De Cooman, 2005). Below are the conceptual frameworks for the relationship between possibility and probability measures.

Aspect	Probability	Possibility
Meaning	Likelihood or frequency	Plausibility or compatibility
Nature of uncertainty	Random (aleatory)	Vague (epistemic)
Additivity	Additive	Maxitive
Operation	Sum (Σ)	Supremum (max)
Data type	Quantitative	Qualitative
Complement relation	$\mathrm{P}(A) + P(A^c) = 1$	$\max[\Pi(A), \Pi(A^c)] = 1$

Figure 2.1: Comparative and integrative model connecting possibility and probability measures through a mapping function

4. RESULTS AD DISCUSSION

4.1 Conceptual Integration

- **Fuzzy Set Representation**

In fuzzy theory, uncertainty is expressed through membership functions, which represent the degree of belongingness to a fuzzy set. In this manner, fuzzy logic generalizes classical logic by allowing truth values to vary continuously between 0 and 1. A membership function defines the degree to which $\pi_A(x)$ belongs to a fuzzy set A. This allows the modeling of gradual transition between true and false, unlike binary logic. Fuzzy measures extend this reasoning by associating uncertainty levels to sets in a non-additive manner. This graded representation enables modeling of uncertainty that is both probabilistic (quantitative) and possibilistic (qualitative) (Zimmermann, 2010).

- **Linking Probability and Possibility via Fuzzy Theory**

Fuzzy theory provides a bridge between probability and possibility through compatibility distributions. Given a probability distribution $\rho(x)$, one can derive a corresponding possibility distribution $\pi(x)$ as:

$$\pi(x) = \min\big(1, c.\rho(x)\big)$$

Conversely, a probability distribution function can be estimated from possibility measures through pignistic transformation which as its origin in plausibility theory:

$$P^*(A) = \sum_{x\epsilon A} \frac{\pi(x)}{\sum_{y\epsilon\Omega}\pi(y)}$$

- **Hybrid Uncertainty Measure**

The integration of both uncertainty measures can be well-defined into a composite measure expressed as:

$$U(A) = \alpha P(A) + (1 - \alpha)\Pi(A)$$

Fuzzy set theory provides a bridge between probability and possibility by expressing uncertainty as a matter of degree of truth rather than a binary state. Within this paradigm, transformations between the two measures are feasible:

Given a probability distribution ρ, one may define a corresponding possibility distribution $\pi(x)$ such that:

$$\Pi(A) = sup_{x \epsilon A}\, \pi(x)$$

Conversely, approximate probabilities can be derived from normalized possibility measures through ranking or defuzzification processes (De Cooman, 2005).

Entropy-based extensions such as De Luca and Termini's fuzzy entropy (1972) generalize Shannon's information measure, allowing the quantification of fuzziness in linguistic or qualitative data.

This integration underpins modern hybrid reasoning systems, which combine statistical inference with fuzzy logic. For example, fuzzy Bayesian networks and possibilistic reasoning models allow reasoning under mixed types of uncertainty—randomness, ambiguity, and incompleteness.

The fuzzy theoretical framework thus extends the mathematical semantics of uncertainty beyond probability, enabling richer and more adaptive representations in AI, risk analysis, and decision support systems.

4.2 Fuzzy Theoretical Implications

- **Probability as an Upper Bound**

In the fuzzy theoretical framework, probability can be viewed as an upper bound for the degree of belief or feasibility associated with an uncertain event. This conceptualization emerges from the observation that probability measures randomness under well-defined conditions, while possibility measures consistency with available knowledge.

Mathematically, for a given event A, the possibility measure $\Pi(A)$ and probability measure $P(A)$ can be related through the inequality:

$$P(A) \leq \Pi(A)$$

Theoretically, this has profound implications. It suggests that probability operates within the limits of possibility: possibility provides the epistemic envelope within which probability-based inference occurs (Dubois & Prade, 1988). In cases of incomplete information, the possibility measure describes the feasible range of probabilistic values.

This relation allows possibility theory to generalize probability theory. When complete information is available, possibility collapses to probability, but under uncertainty or vagueness, possibility remains more expressive. Hence, probability may be viewed as a crisp projection of the more general fuzzy or possibilistic model.

- **Duality as a Measure of Necessity**

The duality between possibility and necessity formalizes the interplay between potential and certainty. Possibility expresses how plausible an event is, while necessity captures how strongly the evidence supports that event.

The dual relationship Is given by:

$$N(A) = 1 - \Pi(A^c)$$

This duality means that high necessity implies high possibility, but not vice versa. For example, if it is necessary that a system fails, it is also possible that it fails; however, if it is merely possible, necessity may still be low.

Conceptually, this dual structure resembles modal logic, where necessity corresponds to the modal operator □ ("it must be so") and possibility corresponds to ◇ ("it may be so"). Within fuzzy logic, these measures quantify the degrees of these modalities (Dubois, 2011).

This duality also reinforces the asymmetry between affirmation and denial in reasoning: while possibility tolerates incomplete support, necessity demands strong and consistent evidence. Therefore, necessity functions as a lower bound of certainty, complementing the upper bound role of probability.

- **Unification under Fuzzy Frameworks**

The unification of probability and possibility within fuzzy set theory represents one of the most conceptually elegant syntheses in modern uncertainty science. Both theories can be interpreted as special cases of fuzzy measures—non-additive set functions that relax the additivity constraint of probability.

A fuzzy measure $m(A)$ satisfies:

$$M(\emptyset) = 0, m(\Omega) = 1, and\ A \subseteq B \implies m(A) \leq m(B)$$

Probability corresponds to additive fuzzy measures.

Possibility corresponds to maxitive fuzzy measures.

Necessity corresponds to minitive fuzzy measures.

This unification reveals that both probability and possibility are manifestations of a broader continuum of uncertainty quantification methods (Zimmermann, 2010). It implies that reasoning systems can move fluidly between probabilistic precision and possibilistic flexibility depending on data availability and interpretive needs.

In applied AI and risk analysis, this theoretical unification enables hybrid models—for instance, fuzzy Bayesian inference, where fuzzy membership functions encode epistemic uncertainty within probabilistic structures (De Cooman, 2005). The result is a system capable of modeling both random variability and vague knowledge in a single cohesive framework.

Thus, the unification of possibility and probability not only resolves philosophical dichotomies between subjectivity and objectivity but also offers a mathematically coherent platform for advanced reasoning under uncertainty.

Table 2: Summary of Theoretical Implications

Theoretical Concept	Core Principle	Mathematical Relation	Implication
Probability as Upper Bound	Probability ≤ Possibility	$P(A) \leq \Pi(A)$	Probability is constrained by feasible epistemic knowledge.
Duality as Necessity	Necessity=1-Possibility(Complement)	$N(A) = 1 - \Pi(A^c)$	Necessity formalizes certainty within possibility space.
Unification	Both are fuzzy measures	Probability: additive, Possibility: maxitive	Enables hybrid modeling across AI, decision theory, and risk analysis.

4.3 Limitations

The transformation depends on the chosen parameters which may require domain-specific calibration. Additionally, empirical validation would need computational simulations on fuzzy data sets.

4.4 Applications

- **Environmental Risk Analysis**

Possibility enhances reasoning in knowledge-based systems, especially when linguistic, incomplete information dominates, such as ecological or environmental systems. Environmental systems, such as water pollution and climate variability, often suffer from incomplete data and linguistic assessments. Hybrid fuzzy-probabilistic models enable more resilient evaluations of risk by incorporating expert opinions with probabilistic measurements (Chen, Lee, & Lin, 2012). For instance, pollutant concentration levels may be probabilistically measured, while expert assessments like "moderately high risk" are modeled possibilistically.

Environmental systems often involve incomplete, imprecise, and time-varying data. Fuzzy-probabilistic risk models combine stochastic variability in natural phenomena (e.g., rainfall) with epistemic uncertainty in pollutant data (Aven, 2015). These models improve the estimation of contamination risks and water quality indices.

- **Engineering and Safety Systems**

In engineering reliability, fuzzy logic enhances probabilistic failure models by accounting for imprecise or subjective data (Aven & Zio, 2018). The resulting hybrid models improve risk prioritization when empirical data are sparse.

Engineering systems frequently rely on expert assessments when precise probabilities are unavailable. Fuzzy reliability analysis integrates possibility distributions with probabilistic safety margins (Chen et al., 2012). This is especially useful in structural health monitoring and fault prediction under incomplete data conditions.

- **Artificial Intelligence and Decision Support**

AI systems frequently encounter ambiguous or incomplete information. Possibility theory enhances knowledge representation, natural language processing, and diagnostic reasoning by capturing linguistic uncertainty, while probability provides statistical grounding (Dubois, 2011). Combining both leads to improved explainable AI systems capable of reasoning under complex uncertainty.

In AI, hybrid models combining Bayesian reasoning and fuzzy inference enhance decision-making under incomplete or linguistic information. Possibility theory supports non-monotonic reasoning, allowing intelligent systems to revise conclusions when new evidence arises.

Overall, the fuzzy theoretical approach broadens the expressive power of uncertainty modeling, providing flexible yet rigorous tools for complex real-world problems.

5. CONCLUSIVE REMARKS

5.1 Discussion

The measures of probability and possibility are not competing paradigms but complementary representations of different uncertainty dimensions. Probability addresses randomness, while possibility models vagueness. Through fuzzy theory, these can coexist within a unified framework that reflects human reasoning—balancing quantitative rigor with qualitative flexibility (Zadeh, 1978; Dubois & Prade, 1988).

Probability and possibility offer complementary paradigms for uncertainty representation. Probability provides a rigorous quantitative basis for stochastic processes, while possibility accommodates qualitative reasoning and imprecision.

The proposed hybrid fuzzy measure supports adaptive reasoning, shifting between probabilistic and possibilistic dominance based on information completeness. This duality enhances robustness and interpretability in AI, environmental modeling, and decision science.

Their integration under fuzzy logic strengthens interpretability and computational tractability in systems where both randomness and vagueness coexist. This duality reflects the evolving nature of scientific reasoning—from purely quantitative prediction to hybrid models that better reflect human-like inference (Dubois, 2011).

5.2 Conclusion

This study establishes a theoretical synthesis of the measures of possibility and probability under a fuzzy logic framework. Probability quantifies random uncertainty, while possibility captures epistemic imprecision. It demonstrates that possibility and probability are not in competition as both approaches offer distinct yet complementary ways of understanding uncertainty. By integrating both, fuzzy theory offers a powerful mechanism for modeling uncertainty in systems characterized by incomplete, vague, or ambiguous information. The fuzzy theoretical framework supports hybrid modeling and adaptive reasoning in science, engineering, and AI, bridging the gap between deterministic models and human judgment. The unified approach enhances decision accuracy, interpretability, and adaptability in uncertain environments - laying the foundation for more human-like reasoning in AI and risk modeling. Future work should explore empirical validation and the extension of this framework to hybrid Fuzzy-Bayesian reasoning systems.

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