Available on: http://gjetr.ep-journals.org/ E-ISSN: 3051-3782, P-ISSN: 3051-3774

Global Journal of Engineering and Technology Review

Volume: 02 Issue: 08 August 2026

Branch-And-Cut and A Sequential Heuristic for Multimodal Multi-Demand Routing

KONE Oumar
Université Nangui Abrogoua, UFR SFA, LMI, Abidjan, Côte d'Ivoire

EDI Y. F. Aristide
Université Nangui Abrogoua, UFR SFA, LMI, Abidjan, Côte d'Ivoire

EDI K. Hilaire
Université Nangui Abrogoua, UFR SFA, LMI, Abidjan, Côte d'Ivoire

TAKOUDA P. L. Matthias
Laurentian University, Sudbury, Ontario, Canada

Published Date: August 29, 2026 **Article DOI : 10.65150/EP-gjetr/V2E8/2026-06**

ABSTRACT: We study multimodal multi-demand routing with coupled capacities, modeled by the compact MILP P_ext. We compare 2 methods (BAC, SEQ) in a scalability study on **1000 instances** (20 groups × 50), measuring CPU time, solution quality, dual certificates and structural size. All heuristics share an **elementary-path** decoder consistent with the MILP objective.

KEYWORDS: multimodal transport; MILP; branch-and-cut; metaheuristics; scalability; operations research

INTRODUCTION

Urban mobility in many metropolitan areas of the Global South is delivered jointly by a formal, scheduled sector (buses, trains, water buses) and an informal, unscheduled sector (shared minibuses and taxis operating on a fill-and-go basis) (Cervero and Golub, 2007; Kumar and Barett, 2008). In cities such as Abidjan, the informal sector alone accounts for the large majority of motorized trips (Gwilliam, 2008), yet it is precisely the sector for which systematic operational data — routes, schedules, ridership, travel-time distributions — are the scarcest. Planners and researchers who wish to design, route, or evaluate services on such networks are therefore confronted with a joint modeling and data problem: the network to be optimized is inherently multimodal, but the calibrated instances needed to test candidate algorithms rarely exist.

Multimodal transport combines several modes on a shared network. When several demands are routed jointly, decisions are coupled by shared capacities and transfer rules. The problem is NP-hard (Wolsey, 1998; Ahuja *et al.*, 1993; Magnanti and Wong, 1984; Crainic, 2000; SteadieSeifi *et al.*, 2014)). This intractability is compounded, in the formal/informal setting, by structural features that do not arise in classical single-sector transit design: sector-specific travel-time laws, cross-sector transfer penalties, and a bound on the number of mode changes allowed along a journey. Any solution method proposed for this class of problem must therefore be evaluated not only on solution quality, but on how its computational behavior scales as network size and modal complexity grow — precisely the regime in which formal/informal multimodal networks in data-scarce metropolitan contexts are expected to sit.

LITERATURE REVIEW

Exact methods: branch-and-cut. Branch-and-cut is a mature paradigm for transit and transportation network design. Marín and García-Ródenas (2009), Gutiérrez-Jarpa *et al.* (2013) and Canca *et al.* (2019) solve integrated rapid-transit design and line-planning models by branch-and-cut inside a commercial solver, validated on real networks in Concepción, Chile. Closer to the present setting, Gao *et al.* (2026) recently proposed the first branch-and-cut algorithm tailored specifically to the transit route network design problem, introducing a dedicated valid inequality and a “transfer cut” that together allow Mandl’s benchmark network to be solved to proven optimality for the first time, with graded experiments extending up to the large Mumford instances. Across this literature, branch-and-cut is the method of choice whenever a certified optimum or a provable gap is required, at the cost of solver time that grows sharply with network size and modal complexity — the trade-off this paper measures directly against a fast constructive alternative.

Constructive heuristics. Greedy sequential construction is the oldest algorithmic idea in transit network design. Baaj and Mahmassani’s hybrid route-generation heuristic (Baaj and Mahmassani, 1991; 1995) established the template — build routes by sequentially extending shortest, highest-demand paths — that essentially every subsequent metaheuristic in this literature uses as an initializer. Zarrinmehr *et al.* (2016) refined this idea with the Route Generation Algorithm and its extended variant, restricting candidate insertions to locally-adjacent nodes to control the computational cost of scaling to real-size networks. A more recent demand-driven variant (Liu *et al.*, 2022) grows routes stop by stop, greedily selecting at each step the stop that maximizes direct-service coverage against a continuously updated origin–destination demand matrix. Across

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

this literature, greedy construction is valued for its speed and simplicity, but it is rarely benchmarked as a standalone method against an exact reference: it is used almost exclusively to seed a subsequent metaheuristic search rather than to be evaluated on its own time–quality trade-off. This leaves open exactly how large a gap a purely greedy method concedes relative to an exact branch-and-cut algorithm on the same instances — the comparison this paper undertakes directly, without an intervening metaheuristic layer.

The formal/informal gap. None of the exact or constructive-heuristic studies above addresses a network that jointly represents a scheduled formal sector and a stochastic, fill-and-go informal sector. To our knowledge, the only published model that is simultaneously multimodal, explicitly formal-and-informal, and calibrated on a developing-country network is the MILP of Koné *et al.* (2026), itself an extension of the formal-sector-only formulation of Edi *et al.* (2025), both solved with a commercial MILP solver rather than a purpose-built algorithm. No branch-and-cut or greedy construction study — exact or heuristic — has yet been benchmarked on instances that jointly represent formal and informal transport at the arc level. This gap is what the present paper addresses; the broader metaheuristic literature for transit network design (genetic algorithms, tabu search, memetic algorithms), which is likewise absent from the formal/informal setting, is left to a follow-up paper.

Contributions. We address this gap by comparing two methods on P_ext: BAC, a branch-and-cut algorithm applied to a compact formulation and solved with CPLEX, serving as our exact reference; and SEQ, a greedy sequential construction heuristic. Placing these two paradigms side by side — one exact and certifying, the other constructive and fast — lets us ask, for the first time on a formal/informal multimodal network, how much optimality a greedy method concedes, and at what computational cost the exact reference buys its guarantee.

This comparison rests on four contributions. First, we give a compact formulation of P_ext for the multimodal formal/informal routing problem that is aligned directly with our implementation, so that every constraint solved by BAC corresponds one-to-one with a modeling choice made explicit in the paper rather than buried in code. Second, we adapt each method to P_ext carefully rather than borrowing it unchanged from the single-sector literature: both BAC and SEQ are restricted to elementary paths and evaluated against an identical objective, so that the comparison isolates the algorithmic paradigm itself rather than incidental differences in how each method happens to model the problem. Third, we build a reproducible computational campaign of 1,000 instances, generated with a probabilistic instance generator calibrated on formal and informal Abidjan transport data, giving both methods a common, publicly reproducible testbed at a scale no prior formal/informal study has used. Fourth, we report a comparative time–quality–certificate analysis of BAC against SEQ across this campaign, quantifying how the optimality gap conceded by the greedy heuristic, and the runtime required by the exact reference, evolve as network size and modal complexity scale.

To our knowledge, this is the first computational study to place an exact branch-and-cut algorithm and a greedy sequential heuristic side by side on a multimodal network that explicitly integrates formal and informal transport, closing the specific methodological gap identified above.

MATERIALS AND METHODS

Problem and formulation

We consider the multi-demand multimodal routing problem with coupled capacities on a directed network. It is modelled by the compact mixed-integer linear program denoted P_ext, aligned with the implementation used in the experiments (CPLEX / docplex solver).

Sets and parameters

Let N be the set of nodes and $A \subseteq N \times N$ the set of physical arcs. For each arc $(i,j) \in A$, let M_{ij} be the finite, non-empty set of modes (transport modes) usable on (i,j). We call service any triple (i,j,m) with $m \in M_{ij}$. Let K be the set of demands. Each demand $k \in K$ has an origin $o^k \in N$, a destination $d_k \in N$, an integer load $q_k \in Z+$, and an integer R_k^{max} bounding the number of mode changes (transfers) along the route of k. The residual capacities on services are given by $u_{ijm} \in Z+$ (maximum volume that may transit on (i,j,m)). The unit marginal costs are $c_{ijm}^k \geq 0$ (per unit of load for demand k on service (i,j,m)). The fixed activation costs $f_{ijm} \geq 0$ apply whenever service (i,j,m) is used by at least one demand. For each node $s \in N$, a relation $\tau_s \subseteq M_s \times M_s$ specifies the mode pairs (m_1, m_2) allowed for a transfer at s (M_s: modes incident to s). Transfer costs $\gamma_{sm1m2} \geq 0$ and, where applicable, additional terms tied to sector attributes σ_{sm1m2} and δ^k may be incurred when a transfer is performed (implementation: $\gamma_{sm1m2} + \delta^k \sigma_{sm1m2}$ per activated transfer variable).

Decision variables

For each demand k, each arc (i, j) ∈ A and each mode $m \in M_{ij}$, we define routing indicators

$$x_{ijm}^k \in \{0,1\}, \tag{1}$$

equal to 1 if demand k uses service (i,j,m). The routing is a unit path per demand (a single unit of flow from o^k to d^k).

For each service (i,j,m),

$$y_{ijm} \in \{0,1\} \tag{2}$$

indicates whether the service is activated (used by at least one demand).

For each intermediate node $s \in N \setminus \{o^k, d^k\}$, and each pair $(m_1, m_2) \in \tau_s$ with $m_1 \neq m_2$ for which transfer variables are generated in the model,

$$t_{sm_1m_2}^k \in \{0,1\} \tag{3}$$

encodes a transfer from mode m_1 to mode m_2 at node s for demand k.

2.3 Objective function

The objective is to minimise the total cost (denoted Z):

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

$$\min\ Z=\sum_{k\in\mathcal{K}}\sum_{(i,j)\in A}\sum_{m\in M_{ij}} q^k c^k_{ijm} x^k_{ijm} + \sum_{(i,j)\in A}\sum_{m\in M_{ij}} f_{ijm} y_{ijm} + \sum_{k,s,m_1,m_2} (\gamma_{sm_1m_2} + \delta^k \sigma_{m_1m_2}) t^k_{sm_1m_2} \tag{4}$$

where the last sum ranges over the transfer variables actually created (pairs allowed by τ_s, nodes s distinct from o^k and d^k). The coefficients σ and δ are zero in instances where these terms are not used.

2.4 Constraints

At most one mode per physical arc and per demand. For all $(i,j)\ \in\ A$ and all $k\ \in\ K$:

$$\sum_{m\in M_{ij}} x^k_{ijm} \le 1 \tag{5}$$

Flow conservation (unit path). For all $k\ \in\ K$ and all $v\ \in\ N$:

$$\sum_{\substack{(i,j,m):i=v\\(i,j)\in A, m\in M_{ij}}} x^k_{ijm} - \sum_{\substack{(i,j,m):j=v\\(i,j)\in A, m\in M_{ij}}} x^k_{ijm} = \begin{cases} 1 & \text{if } v=o^k \\ -1 & \text{if } v=d^k \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Coupled capacities. For every service (i,j,m):

$$\sum_{k\in\mathcal{K}} q^k x^k_{ijm} \le u_{ijm} \tag{7}$$

Activation–usage linking. For every (i,j,m) and every k:

$$y_{ijm} \ge x^k_{ijm}, y_{ijm} \le \sum_{k\in\mathcal{K}} x^k_{ijm} \tag{8}$$

Thus $y_{ijm}\ = 1$ if and only if at least one x^k_{ijm} equals 1.

Transfers and forbidden pairs. For each variable $t^k_{sm_1m_2}$, linear constraints link t to the inbound flow in mode m_1 and the outbound flow in mode m_2 at s (logical AND-type constraints that activate a transfer only if the incident arcs match); pairs (m_1, m_2) not listed in τ_s are forbidden by constraints preventing the simultaneous use of an unauthorised inbound m_1 and outbound m_2.

Bound on the number of transfers. For all $k\ \in\ K$:

$$\sum_{s,m_1,m_2} t^k_{sm_1m_2} \le R^k_{max} \tag{9}$$

The sum ranges over the transfer variables defined for k (possibly with a relaxation of the bound in some numerical variants to avoid artificial infeasibility for very small values of R^k_{max}).

Remark 1 (Link with the implementation). The formulation above corresponds to the MILP model implemented in the source code: order of summations, generation of the t variables only for the transfers listed in τ_s, and objective coefficients for transfers. For the full algebraic detail of the constraints on the t variables and of the forbidden pairs, we refer to the commented listing of the MILP module and to the companion article Oumar *et al.* (2026).

Remark 2 (Complexity). The multi-commodity routing problem with fixed costs and design constraints is NP-hard in general; practical solution relies on the exact or approximate methods discussed in the following sections, depending on instance size and time budget.

3.1 Branch-and-cut (BAC)

Branch-and-cut solves the compact P_ext model with CPLEX (presolve, cuts, SEQ MIP start). Under tolerance ε, it returns Z*, a dual bound and a gap certificate. BAC is the ground truth.

3.2 Sequential heuristic (SEQ)

Demands are ordered by decreasing load q^k and routed greedily via an elementary residual-capacity shortest path, accounting for already paid fixed costs. Extremely fast feasible primal.

3.3 Pseudocode

The pseudocodes of the solutions approach Greedy sequential heuristic (SEQ) (Algorithm 1) is provided next.

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

Algorithm 1. Greedy sequential heuristic (SEQ)

Require: Instance P_ext; base order (by decreasing q^k); number of starts S.
Ensure: Feasible solution (paths) and cost Z.
Z* ← +∞
for s = 1 to S do
  Build an order π (base, then random permutations if $s > 1$)
  Residual capacities û ← u; activated services $A \leftarrow \emptyset$; $Z \leftarrow 0$
  for each demand k in order π **do**
    Shortest pathp^k from o^k to d^k under û, bound R^k_{max}, zero fixed cost on A
    if no feasible path **then**
      move to next start
      end if
    $Z \leftarrow Z + cost(p^k)$; update û (subtract q^k) and A
  end for
  if Z < Z* **then**
    Z* ← Z; store the paths
  end if
end for
return solution of cost Z*

Experimental protocol

Instances are generated parametrically ($\alpha = 2, |M| = 5$), in 20 groups of 50 (1000 total) of increasing size and variable density. Fixed settings were set and the optimality gap Δ_{opt} was relative to BAC.

RESULTS

The results of our numerical experiments are summarized in Table 1, 2 and 3, and Figure 1, 2 and 3.

Table 1. ***Global synthesis over 1000 instances.***

Method	$\bar{t}\ (s)$	med. t (s)	max t (s)	Z	Δ_{opt} (%)	% opt.
BAC	0.65	0.54	2.3	90.32	0.000	100.0
SEQ	0.01	0.01	0.1	91.07	0.372	0.0

Table 2. Structure and mean CPU time by group (groups 1–20).

Gr.	|V|	|A|	|K|	Vars	Cons.	BAC (s)	SEQ (s)
1	10	54	2	646	1334	0.18	0.00
2	10	53	2	621	1283	0.16	0.00
3	10	55	2	649	1340	0.17	0.00
4	12	76	2	833	1715	0.23	0.00
5	12	76	3	1197	2477	0.35	0.00
6	12	75	3	1167	2403	0.33	0.00
7	12	77	3	1206	2496	0.37	0.00
8	12	76	3	1188	2447	0.35	0.00
9	14	101	3	1515	3115	0.48	0.01
10	14	102	3	1518	3116	0.48	0.01
11	14	100	4	1944	4016	0.63	0.01
12	14	103	4	1988	4099	0.65	0.01
13	16	129	4	2352	4817	0.82	0.01
14	16	133	4	2447	5011	0.89	0.01
15	16	132	4	2422	4955	0.84	0.01
16	16	133	4	2461	5034	0.88	0.01
17	16	127	5	2824	5789	0.99	0.01
18	18	164	5	3560	7276	1.41	0.02

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

19	18	165	5	3545	7226	1.41	0.02
20	18	166	5	3597	7330	1.47	0.02

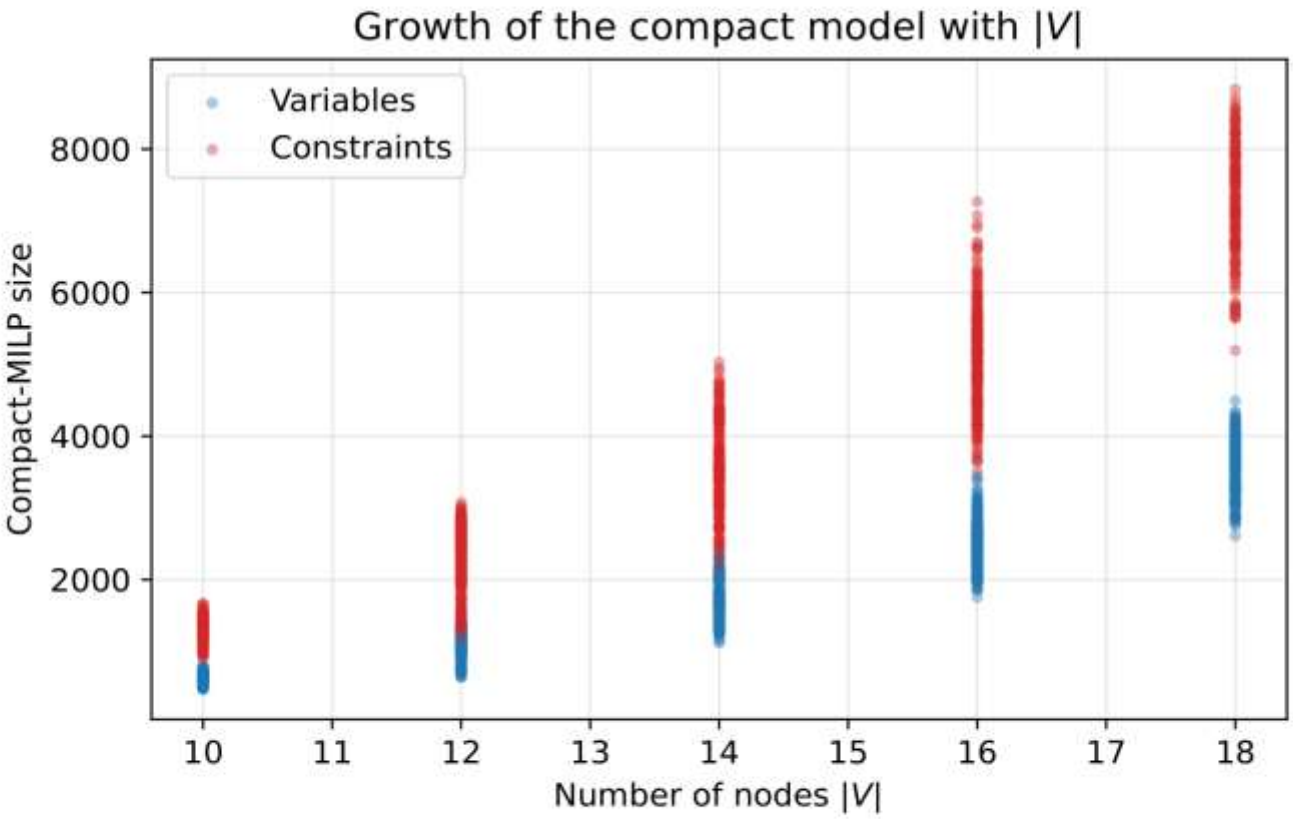

Figure 1. Growth of the compact MILP with |V|.

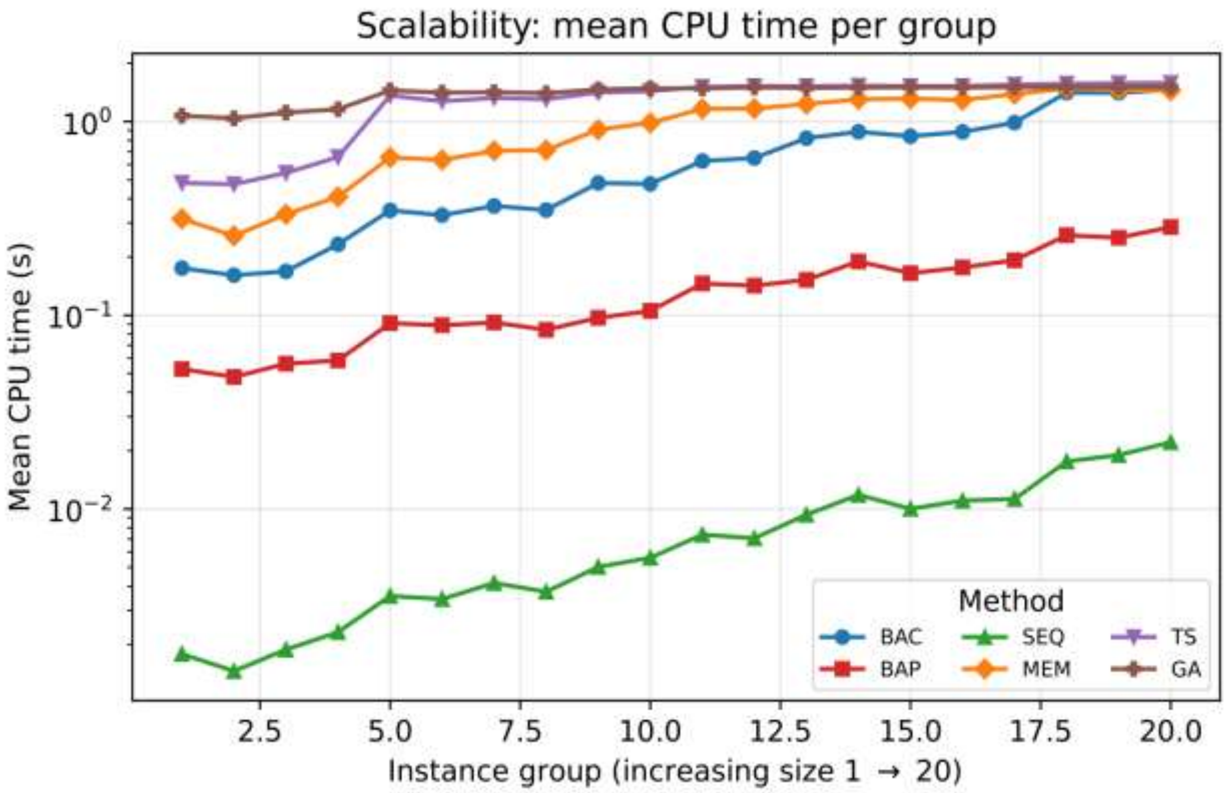

Figure 2. Mean CPU time by group (log).

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

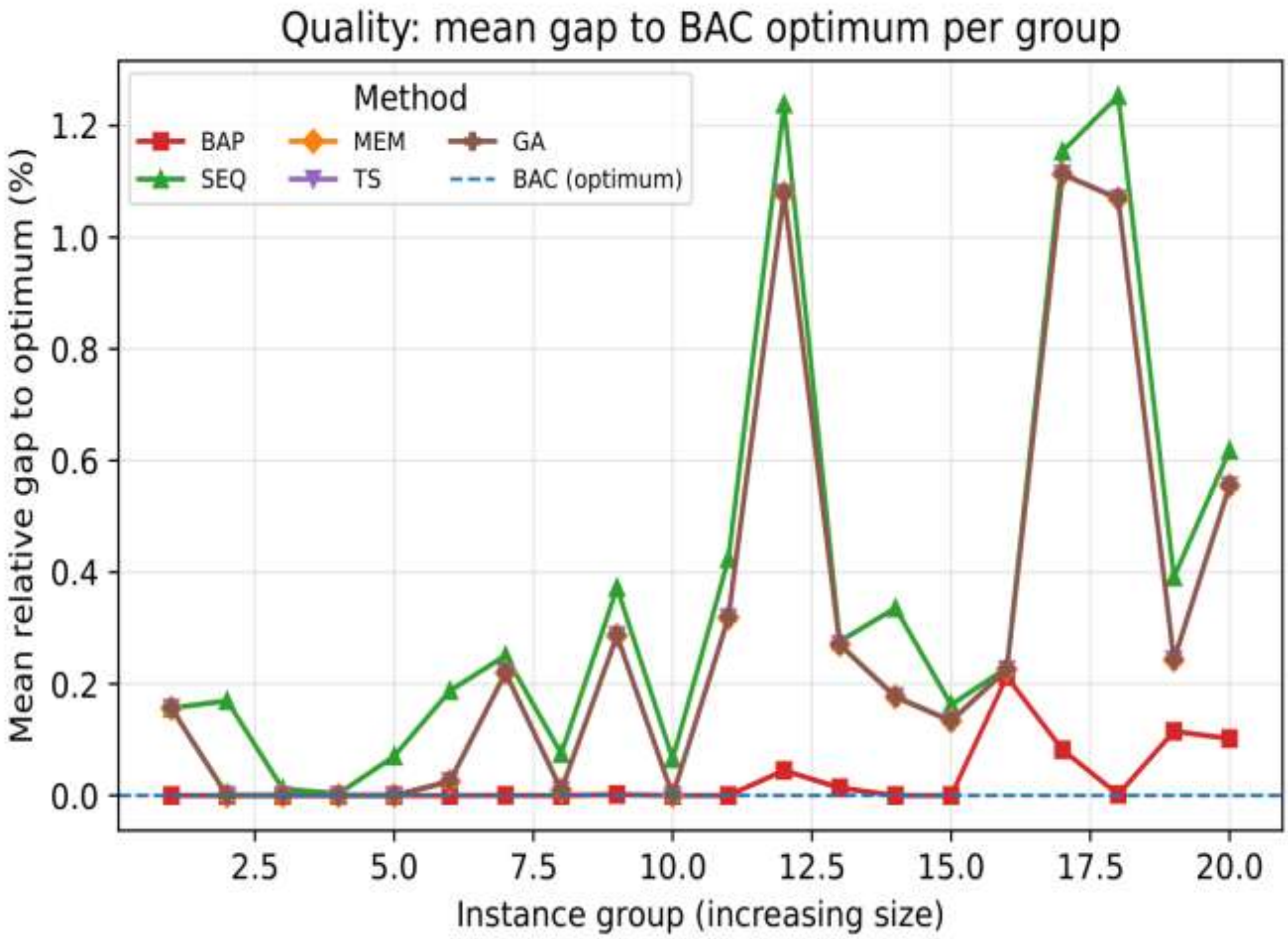

Figure 3. Mean gap to BAC optimum.

Table 3. Mean gaps (%): SEQ.

Gr.	Δ_SEQ (%)
1	0.157
2	0.170
3	0.012
4	0.003
5	0.070
6	0.188
7	0.251
8	0.075
9	0.372
10	0.066
11	0.424
12	1.239
13	0.276
14	0.336
15	0.163
16	0.229
17	1.154

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

18	1.253
19	0.391
20	0.618

DISCUSSION

Over 1000 instances, methods BAC, SEQ show complementary profiles (Table 1):

- BAC: mean t = 0.654 s, mean Δ_opt = 0.000%, optimal status 100.0%.
- SEQ: mean t = 0.008 s, mean Δ_opt = 0.372%.

We observe primal consistency (heuristics ≥ Z*). Operational recommendations depend on the time budget and certificate needs.

CONCLUSION

We compared 2 methods (BAC, SEQ) on P_ext over 1000 instances. Time–quality–certificate profiles are complementary; a cooperative pipeline is suggested. Future work includes developing and comparing metaheuristics approaches, as well as a full branch-and-price algorithms, dual stabilization and experimentations on larger instances.

ACKNOWLEDGMENT

Not applicable.

REFERENCES

1) Ahuja, R. K., Magnanti, T. L., & Orlin, J. B. (1993). *Network flows: Theory, algorithms, and applications*. Prentice Hall.
2) Baaj, M. H., & Mahmassani, H. S. (1991). An AI-based approach for transit route system planning and design. *Journal of Advanced Transportation, 25*(2), 187–209.
3) Baaj, M. H., & Mahmassani, H. S. (1995). Hybrid route generation heuristic algorithm for the design of transit networks. *Transportation Research Part C: Emerging Technologies, 3*(1), 31–50.
4) Barnhart, C., Hane, C. A., & Johnson, E. L. (1998). Branch-and-price: Column generation for solving huge integer programs. *Operations Research, 46*(3), 316–329.
5) Blum, C., & Roli, A. (2003). Metaheuristics in combinatorial optimization: Overview and conceptual comparison. *ACM Computing Surveys, 35*(3), 268–308.
6) Canca, D., De-Los-Santos, A., Laporte, G., & Mesa, J. A. (2019). Integrated railway rapid transit network design and line planning problem with maximum profit. *Transportation Research Part E: Logistics and Transportation Review, 127*, 1–30.
7) Cervero, R., & Golub, A. (2007). Informal transport: A global perspective. *Transport Policy, 14*(6), 445–457.
8) Crainic, T. G. (2000). Service network design in freight transportation. *European Journal of Operational Research, 122*(2), 272–288.
9) Crainic, T. G., & Rousseau, J.-M. (1986). Multicommodity, multimode freight transportation: A general modeling and algorithmic framework for the service network design problem. *Transportation Research Part B: Methodological, 20*(3), 225–242.
10) Desaulniers, G., Desrosiers, J., & Solomon, M. M. (Eds.). (2005). *Column generation*. Springer.
11) Dolan, E. D., & Moré, J. J. (2002). Benchmarking optimization software with performance profiles. *Mathematical Programming, 91*(2), 201–213.
12) Edi, Y. F. A., Koné, O., & Edi, K. H. (2025). A new mixed-line programming approach to the problem of multimodal urban transit. *Statistics, Optimization & Information Computing, 14*(3), 1458–1472.
13) Gao, W., Ma, J., He, Y., & Dong, C. (2026). An exact approach for the transit route network design problem. *European Journal of Operational Research, 334*(1), 111–127.
14) Glover, F. (1989). Tabu search—Part I. *ORSA Journal on Computing, 1*(3), 190–206.
15) Glover, F., & Laguna, M. (1997). *Tabu search*. Kluwer Academic Publishers.
16) Goldberg, D. E. (1989). *Genetic algorithms in search, optimization, and machine learning*. Addison-Wesley.
17) Gwilliam, K. (2008). Public transport in developing countries. *Transport Reviews, 28*(4), 415–432.
18) Holland, J. H. (1975). *Adaptation in natural and artificial systems*. University of Michigan Press.
19) Koné, O., Edi, Y. F. A., Edi, K. H., & Takouda, P. M. L. (2026). MILP for multimodal urban transport: Formal and informal sectors. *International Journal of Advanced Computer Science and Applications, 17*(5).
20) Kumar, A., & Barrett, F. (2008). *Stuck in traffic: Urban transport in Africa* (AICD Background Paper No. 1). World Bank.
21) Liu, Y., Zhang, H., Xu, T., & Chen, Y. (2022). A heuristic algorithm based on travel demand for transit network design. *Sustainability, 14*(17), Article 11097.
22) Lübbecke, M. E., & Desrosiers, J. (2005). Selected topics in column generation. *Operations Research, 53*(6), 1007–1023.
23) Magnanti, T. L., & Wong, R. T. (1984). Network design and transportation planning: Models and algorithms. *Transportation Science, 18*(1), 1–55.
24) Gutiérrez-Jarpa, G., Obreque, C., Laporte, G., & Marianov, V. (2013). Rapid transit network design for optimal cost and origin–destination demand capture. *Computers & Operations Research, 40*(12), 3000–3009.
25) Marín, Á., & García-Ródenas, R. (2009). Location of infrastructure in urban railway networks. *Computers & Operations Research, 36*(5), 1461–1477.

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/

26) Moscato, P. (1999). Memetic algorithms: A short introduction. In *New ideas in optimization* (pp. 219–234). McGraw-Hill.
27) SteadieSeifi, M., Dellaert, N. P., Nuijten, W., Van Woensel, T., & Raoufi, R. (2014). Multimodal freight transportation planning: A literature review. *European Journal of Operational Research, 233*(1), 1–15.
28) Wolsey, L. A. (1998). *Integer programming*. Wiley.
29) Zarrinmehr, A., Saffarzadeh, M., Seyedabrishami, S., & Nie, Y. M. (2016). A path-based greedy algorithm for multi-objective transit routes design with elastic demand. *Public Transport, 8*(2), 261–293.

License: This is an open access article under the CC BY 4.0 license: https://creativecommons.org/licenses/by/4.0/